RESULT 7: UNDER ASSUMPTION BLOCKS 1-6, THE FOLLOWING “REHABILITATED” VERSIONS OF MARX’S RESULTS ON THE TRANSFORMATION PROBLEM ALL HOLD:

(I) THE AGGREGATE MARKET VALUE OF THE BALANCED GROWTH EQUILIBRIUM OUTPUTS MEASURED IN MARXIAN PRICES IS EQUAL TO THEIR AGGREGATE LABOR VALUE

(II) UNDER LINEAR DEPENDENCE OF INDUSTRIES, THE COST-PRICE OF A COMMODITY IN TERMS OF MARXIAN PRICES IS LESS THAN OR EQUAL TO ITS LABOR VALUE, WITH STRICT EQUALITY FOR ANY GOOD THAT REQUIRES A POSITIVE AMOUNT OF LABOR.

(III) UNDER LINEAR DEPENDENCE OF INDUSTRIES, THE TOTAL SURPLUS VALUE ACROSS ALL GOODS EQUALS TOTAL PROFITS MEASURED IN MARXIAN PRICES IN THE STATE OF BALANCED GROWTH.

(IV) A) THE MARXIAN PRICE OF A COMMODITY EQUALS ITS LABOR VALUE ONLY IN THE SPHERE IN WHICH THE VALUE COMPOSITION OF CAPITAL EQUALS THE SOCIAL AVERAGE (DEFINED AS THE WEIGHTED AVERAGE OF THE VALUE COMPOSITIONS ACROSS ALL INDUSTRIES WHERE THE WEIGHTS ARE INDUSTRY SHARES OF THE AGGREGATE VARIABLE CAPITAL IN BALANCED GROWTH EQUILIBRIUM). B) UNDER LINEAR DEPENDENCE OF INDUSTRIES, THE PRICES OF COMMODITIES ARE PROPORTIONAL TO THEIR LABOR VALUES ONLY IN SPHERES IN WHICH THE VALUE COMPOSITION OF CAPITAL IS THE SAME.

(V) A) THE VALUE OF COMMODITIES PRODUCED BY CAPITAL OF HIGHER (LOWER) VALUE COMPOSITION THAN THE SOCIAL AVERAGE IS SMALLER (LARGER) THAN THEIR MARXIAN PRICE OF PRODUCTION B) UNDER LINEAR DEPENDENCE OF INDUSTRIES, THE RELATIVE PRICE OF COMMODITY \(i\) COMPARED TO \(j\) IS GREATER (LESS) THAN THE RELATIVE VALUE OF \(i\) TO \(j\) IF \(i\) IS HIGHER (LOWER) THAN \(j\) IN THE VALUE COMPOSITION OF CAPITAL.

To prove Proposition (I), recall that by definition the Marxian price \(q_i\) of good \(i\) is given by:

\[ q_i = (1 + \pi)(C_i + V_i), \tag{1} \]

where \(\pi\) is the equilibrium rate of profit (which exists, is unique, and is strictly positive under Assumption Blocks 1-6), \(C_i\) is the constant capital of good \(i\), and \(V_i\) is the variable capital of good \(i\). Recall from Corollary 2 to the Morishima-Seton Equation in “Morishima’s Results for Capitalist Production” that under Assumption Blocks 1-5 we can express the rate of profit as:

\[ \pi = \frac{\sum_{j=1}^{m} S_j y_j}{\sum_{j=1}^{m} C_j y_j + \sum_{j=1}^{m} V_j y_j}, \tag{2a} \]

\[ = \frac{\sum_{j=1}^{m} S_j y_j}{\sum_{j=1}^{m} (C_j + V_j)y_j} \tag{2b} \]
where $S_j$, $C_j$, and $V_j$ are the surplus value, constant capital, and variable capital, respectively, of good $j$, and $y_j$ is the amount of good $j$ in the characteristic, or balanced growth equilibrium, vector, for $j = 1, \ldots, m$. Plugging (2b) into (1) and using the decomposition of labor value (Result 2 of “Morishima’s Results for Capitalist Production”) yields:

\[
q_i = \left(1 + \frac{\sum_{j=1}^m s_j y_j}{\sum_{j=1}^m (c_j + v_j) y_j}\right) (C_i + V_i) \tag{3a}
\]

\[
= \left(\frac{\sum_{j=1}^m (c_j + v_j + s_j) y_j}{\sum_{j=1}^m (c_j + v_j) y_j}\right) (C_i + V_i) \tag{3b}
\]

\[
= \left(\frac{\sum_{j=1}^m \lambda_j y_j}{\sum_{j=1}^m (c_j + v_j) y_j}\right) (C_i + V_i), \tag{3c}
\]

where $\lambda_j$ is the labor value of good $j$. Multiplying both sides of (3c) by $y_i$, summing across all goods $i = 1, \ldots, m$, and recognizing that the index of summation can be switched from $i$ to $j$ as desired, yields:

\[
q_i y_i = \left(\frac{\sum_{j=1}^m \lambda_j y_j}{\sum_{j=1}^m (c_j + v_j) y_j}\right) (C_i + V_i) y_i \tag{4a}
\]

\[
\Rightarrow \sum_{i=1}^m q_i y_i = \left(\frac{\sum_{j=1}^m \lambda_j y_j}{\sum_{j=1}^m (c_j + v_j) y_j}\right) \sum_{i=1}^m (C_i + V_i) y_i \tag{4b}
\]

\[
\Rightarrow \sum_{i=1}^m q_i y_i = \sum_{j=1}^m \lambda_j y_j \left(\frac{\sum_{i=1}^m (C_i + V_i) y_i}{\sum_{j=1}^m (c_j + v_j) y_j}\right) \tag{4c}
\]

\[
\Rightarrow \sum_{j=1}^m q_j y_j = \sum_{j=1}^m \lambda_j y_j \left(\frac{\sum_{i=1}^m (C_i + V_i) y_i}{\sum_{j=1}^m (c_j + v_j) y_j}\right) \tag{4d}
\]

\[
\Rightarrow \sum_{j=1}^m q_j y_j = \sum_{j=1}^m \lambda_j y_j, \tag{4e}
\]

which proves the result since the left side of (4e) is the aggregate market value of the balanced growth bundle measured in Marxian prices and the right side is the aggregate labor value of this bundle.

To prove Proposition (II), assume linear dependence of industries holds. Then by Result 5, the Marxian prices are proportional to the true equilibrium prices. By Result 6, the total capital of each good measured with labor values $(C_i + V_i)$ is equal to the total capital measured with Marxian prices $(C_i^q + V_i^q)$. Therefore, using the decomposition of labor value, we have:

\[
\lambda_i = C_i + V_i + S_i \tag{5a}
\]

\[
\geq C_i + V_i \tag{5b}
\]

\[
= C_i^q + V_i^q, \tag{5c}
\]

where we have used the fact that $S_i = eV_i \geq 0$, where $e$ is the rate of exploitation, in moving from (5a) to (5b), and the equality of $C_i + V_i$ with $C_i^q + V_i^q$ in moving from (5b) to (5c). Note that since $\pi > 0$ under Assumption Blocks 1-6, it follows from the Fundamental Marxian Theorem that $e > 0$. Hence, the inequality in (5b) is strict whenever $V_i > 0$. But $V_i = \omega \Lambda B l_i$, with $\omega = 1/T > 0$ since $T$ is the length of the work day, $\Lambda > 0$ since it is the vector of labor values of wage and luxury goods, all of which are strictly positive under Assumption Blocks 1-3.
(see Result 6 of “Morishima’s Labor Theory of Value Results”), and \( \Lambda_{tj} B > 0 \) since \( \Lambda_{tj} > 0 \) and the subsistence vector \( B \) has at least one strictly positive entry under Assumption Block 4. Therefore, \( V_t > 0 \) if and only if \( l_t > 0 \), so the inequality in (5b) is strict and hence \( \lambda_i > C_i^q + V_i^q \) whenever \( l_t > 0 \). This proves the result.

To prove Proposition (III), assume linear dependence of industries holds. It follows from Result 5 that the Marxian prices are proportional to the true equilibrium prices. Note that the profit per unit in industry \( j \), measured in Marxian prices, would be the Marxian price \( q_j \) minus the constant capital measured in Marxian prices \( C_j^q \) minus the variable capital in Marxian prices \( V_j^q \):

\[
\Pi_j^q = q_j - C_j^q - V_j^q
\]  

(6)

Multiplying (6) through by the balanced growth quantity \( y_j \) of good \( j \) and summing over all industries \( j \) gives the aggregate profit in the state of balanced growth as:

\[
\sum_{j=1}^{m} \Pi_j^q y_j = \sum_{j=1}^{m} (q_j - C_j^q - V_j^q) y_j = \sum_{j=1}^{m} q_j y_j - \sum_{j=1}^{m} (C_j^q + V_j^q) y_j = \sum_{j=1}^{m} (\lambda_j - C_j - V_j) y_j = \sum_{j=1}^{m} S_j y_j,
\]  

(7a)

(7b)

(7c)

(7d)

(7e)

where we have used Result 6 (since Marxian prices are proportional to true equilibrium prices, total capital is preserved under conversion from labor values to Marxian prices) and Proposition 1 (aggregate value of the balanced growth vector in Marxian prices is equal to its aggregate labor value) in moving from (7b) to (7c), and the decomposition of value in moving from (7d) to (7e). Since we have shown that the aggregate profit of the balanced growth vector in Marxian prices is equal to the aggregate surplus value, the result is proved.

To prove Proposition (IV.A), let \( \bar{k} \) be the social average value composition in balanced growth equilibrium, i.e.,

\[
\bar{k} = \frac{\sum_{i=1}^{m} c_i y_i}{\sum_{i=1}^{m} V_i y_i}
\]  

(8)

Observe that as long as no industry has zero variable capital, \( \bar{k} \) can alternatively be written in the form:

\[
\bar{k} = \sum_{i=1}^{m} \left( \frac{V_i y_i}{\sum_{i=1}^{m} V_i y_i} \right) \frac{c_i}{V_i}
\]  

(9)

which shows that \( \bar{k} \) is the weighted average of all the \( C_i/V_i \) values, where the weight for industry \( i \) is that industry’s share in the total variable capital across all industries in balanced growth equilibrium.
Recall from the Morishima-Seton equation that the equilibrium rate of profit $\pi$ satisfies:

$$\pi = \frac{e^{\sum_{i=1}^{m} V_i y_i}}{\sum_{i=1}^{m} c_i y_i + \sum_{i=1}^{m} V_i y_i}$$ (10a)  

$$= \frac{e^{\sum_{i=1}^{m} c_i y_i}}{\sum_{i=1}^{m} y_i y_i + \sum_{i=1}^{m} V_i y_i}$$ (10b)  

$$= \frac{e}{k+1}$$ (10c)

Note that, since $\pi > 0$ under Assumption Blocks 1-6, we also have $e > 0$ by the Fundamental Marxian Theorem. Furthermore, since $C_i > 0$ for all $i$ under Assumption Blocks 1-3, and the characteristic vector $y$ is non-negative and non-zero, it follows that $\sum_{i=1}^{m} c_i y_i > 0$. Since $V_i \geq 0$ for all $i$, we also have $\sum_{i=1}^{m} V_i y_i \geq 0$. Therefore, it follows from (10a) that $\sum_{i=1}^{m} V_i y_i$ cannot be 0, for otherwise $\pi$ would be 0. We can then conclude that $k$ is a finite positive number from (8).

Any industry with zero surplus value would then have zero variable capital (since $S_i = eV_i$) and an infinite value composition of capital (since $k_i = C_i/V_i$), so it could not have a composition of capital equal to the social average.

Using the definition of the Marxian price $q_i$, plugging in (10c), and using $C_i = k_i V_i$ and $S_i = eV_i$, we get:

$$q_i = (1 + \pi)(C_i + V_i)$$ (11a)  

$$= (1 + \frac{e}{k+1})(C_i + V_i)$$ (11b)  

$$= (C_i + V_i) + \frac{e}{k+1}(C_i + V_i)$$ (11c)  

$$= (C_i + V_i) + \frac{e}{k+1} C_i + \frac{e}{k+1} V_i$$ (11d)  

$$= (C_i + V_i) + \frac{e}{k+1} V_i + \frac{e}{k+1} S_i$$ (11e)  

$$= (C_i + V_i) + \frac{e}{k+1} V_i + \frac{k_i S_i}{k+1}$$ (11f)  

$$= (C_i + V_i) + \frac{(k_i+1) S_i}{k+1}$$ (11g)  

$$= (C_i + V_i) + \frac{(k_i+1) S_i}{k+1}$$ (11h)

Since $\lambda_i = C_i + V_i + S_i$, it follows from (11h) that $q_i = \lambda_i$ if and only if:

$$C_i + V_i + \frac{(k_i+1) S_i}{k+1} = C_i + V_i + S_i$$ (12a)  

$$\Leftrightarrow \frac{(k_i+1) S_i}{k+1} = S_i$$ (12b)  

$$\Leftrightarrow (k_i + 1) S_i = S_i(k + 1)$$ (12c)

Note that if $S_i = 0$ then the equation $q_i = \lambda_i$ would imply $(1 + \pi)(C_i + V_i) = (C_i + V_i)$, which is equivalent to $\pi(C_i + V_i) = 0$, which would imply $\pi = 0$ since $C_i > 0$ and $V_i = 0$. But this would contradict the fact that $\pi > 0$ under Assumption Blocks 1-6. Thus, we can assume $S_i > 0$ in (12c) and cancel it from both sides, which yields:
\[ q_i = \lambda_i \]  
\[ \Leftrightarrow (k_i + 1) = (\bar{k} + 1) \]  
\[ \Leftrightarrow k_i = \bar{k} \]  

Thus, we have shown that the Marxian price equals the labor value if and only if the value composition of capital in industry \( i \) is equal to social average, which proves Proposition (IV.A).

To prove (IV.B), assume linear dependence of industries holds. Then by Result 5, the Marxian prices \( q_i \) are proportional to the true equilibrium prices \( p_i \); i.e., there exists a number \( \alpha \) such that \( q_i = \alpha p_i \) for all \( i \). In fact, we know that \( p_i > 0 \) for all \( i \) since all industries earn positive profits under Assumption Blocks 1-6, and \( q_i > 0 \) for all \( i \) since \( C_i > 0 \) and \( V_i \geq 0 \) under Assumption Blocks 1-3, so it follows that \( \alpha > 0 \). Therefore, for all goods \( i \) and \( j \) we have:

\[
\frac{q_i}{q_j} = \frac{\alpha p_i}{\alpha p_j} = \frac{p_i}{p_j} \quad \text{(14a)}
\]

Note that for any goods \( i \) and \( j \) with positive variable capitals, we have from the definition of the Marxian prices and the fact that the value composition \( k_i \) of each industry \( i \) satisfies \( C_i = k_i V_i \) that:

\[
\frac{q_i}{q_j} = \frac{(1+\pi)(C_i+V_i)}{(1+\pi)(C_j+V_j)} = \frac{C_i+V_i}{C_j+V_j} = \frac{k_iV_i+V_i}{k_jV_j+V_j} = \frac{(k_i+1)V_i}{(k_j+1)V_j} \quad \text{(15a)}
\]

From (14b) it then follows that:

\[
\frac{p_i}{p_j} = \frac{(k_i+1)V_i}{(k_j+1)V_j} \quad \text{(16)}
\]

From the decomposition of value formula \( \lambda_i = C_i + V_i + S_i \) and the relations \( C_i = k_i V_i \) and \( S_i = eV_i \) we have:

\[
\frac{\lambda_i}{\lambda_j} = \frac{C_i+V_i+S_i}{C_j+V_j+S_j} = \frac{k_iV_i+V_i+eV_i}{k_jV_j+V_j+eV_j} = \frac{(k_i+1+e)V_i}{(k_j+1+e)V_j} \quad \text{(17a)}
\]
Now suppose \( k_i = k_j \) for two industries \( i \) and \( j \) with positive variable capitals. Then (16) implies that \( p_i/p_j = V_i/V_j \) and (17c) implies that \( \lambda_i/\lambda_j = V_i/V_j \). Hence \( p_i/p_j = \lambda_i/\lambda_j \). But since we can always write \( p_i = \beta_i \lambda_i \) and \( p_j = \beta_j \lambda_j \) for constants \( \beta_i, \beta_j > 0 \) (recall that all prices are positive since all industries earn positive profits under Assumption Blocks 1-6 and all labor values are strictly positive under Assumption Blocks 1-3), we have \( p_i/p_j = \beta_i \lambda_i/\beta_j \lambda_j \). But then \( \lambda_i/\lambda_j = \beta_i/\beta_j \), which implies \( \beta_i/\beta_j = 1 \) since we can cancel \( \lambda_i/\lambda_j \) on both sides (this quantity being strictly positive). Then \( \beta_i = \beta_j \) and we can denote this common magnitude by \( \beta \). We then have \( p_i = \beta \lambda_i \) and \( p_j = \beta \lambda_j \), i.e., prices are proportional to labor values. Now suppose that \( k_i = k_j = \infty \). Then \( V_i = V_j = 0 \), so \( q_i/q_j = C_i/C_j \), which means \( p_i/p_j = C_i/C_j \) since equilibrium prices are proportional to Marxian prices. But decomposition of value implies \( \lambda_i/\lambda_j = C_i/C_j \), so we have \( p_i/p_j = \lambda_i/\lambda_j \). By the same argument as that above, we can conclude that prices are proportional to labor values. This shows that any time two industries have the same value composition of capital, whether it is finite or infinite, their prices will be proportional to their labor values.

To show the converse, suppose prices are proportional to labor values for two industries \( i \) and \( j \); i.e., there exists a number \( \beta > 0 \) such that \( p_i = \beta \lambda_i \) and \( p_j = \beta \lambda_j \). Then \( p_i/p_j = \lambda_i/\lambda_j \). But \( p_i/p_j = q_i/q_j \) by (14b), so \( q_i/q_j = \lambda_i/\lambda_j \). If these industries both have positive variable capitals, then from (15d) and (17c) we have:

\[
\frac{(k_i+1)V_i}{(k_j+1)V_j} = \frac{(k_i+1+e)V_i}{(k_j+1+e)V_j}
\]

The ratio \( V_i/V_j \) being strictly positive, we can cancel it from both sides of (18) to get:

\[
\frac{(k_i+1)}{(k_j+1)} = \frac{(k_i+1+e)}{(k_j+1+e)} \quad (19a)
\]

\[
\Rightarrow (k_i + 1)(k_j + 1 + e) = (k_i + 1 + e)(k_j + 1) \quad (19b)
\]

\[
\Rightarrow (k_i + 1)(k_j + 1) + e(k_i + 1) = (k_i + 1)(k_j + 1) + e(k_j + 1) \quad (19c)
\]

\[
\Rightarrow e(k_i + 1) = e(k_j + 1) \quad (19d)
\]

The rate of exploitation being positive by the Fundamental Marxian Theorem (in light of \( \pi > 0 \)), we can cancel it from both sides of (19d) to get:

\[
k_i + 1 = k_j + 1 \quad (20a)
\]

\[
\Rightarrow k_i = k_j \quad (20b)
\]

so that industries \( i \) and \( j \) have the same (finite) value composition whenever their prices are proportional to their labor values and they both have positive variable capitals. If they both have zero variable capitals, then they both have infinite value compositions, so their value compositions are once again the same. So assume their prices are proportional to their labor values even though one of them, say industry \( i \), has zero variable capital (and hence an infinite value composition), while the other one, industry \( j \), has positive variable capital, and hence a finite value composition. Then since their prices are proportional to their labor values we have...
The Marxian price \( p_i / p_j \) is proportional to the equilibrium price \( q_i / q_j \), and since Marxian prices are proportional to equilibrium prices we have \( p_i / p_j = q_i / q_j \). Hence \( q_i / q_j = \lambda_i / \lambda_j \). But observe that since \( V_i = 0 \) and \( V_j > 0 \), we have:

\[
\frac{q_i}{q_j} = \frac{(1+\pi)C_i}{(1+\pi)(C_j+V_j)} = \frac{C_i}{C_j+V_j}
\]

(21a)

and

\[
\frac{\lambda_i}{\lambda_j} = \frac{C_i}{C_j+V_j+S_j}
\]

(21b)

Putting (21b) and (22) together with the fact that \( q_i / q_j = \lambda_i / \lambda_j \) yields:

\[
\frac{c_i}{c_j+V_j+S_j} = \frac{c_i}{c_j+V_j+S_j}
\]

(23a)

\[
\Rightarrow c_i(C_j + V_j + S_j) = c_i(C_j + V_j)
\]

(23b)

\[
\Rightarrow c_i(C_j + V_j) + c_iS_j = c_i(C_j + V_j)
\]

(23c)

\[
\Rightarrow c_iS_j = 0
\]

(23d)

But since \( C_i > 0 \), (23d) implies that \( S_j = 0 \), so that \( eV_j = 0 \), from which we obtain \( e = 0 \) since \( V_j > 0 \). This contradicts the conclusion from the Fundamental Marxian Theorem that \( e > 0 \) since \( \pi > 0 \). We conclude that this case is not possible (prices proportional to labor values for an industry that has positive variable capital and an industry with zero variable capital). So if prices are proportional to labor values, they either both have positive variable capital (and a finite value composition) or they both have zero variable capital (an infinite value composition). Either way, their value compositions are the same. Thus we have shown that whenever two industries have prices that are equiproportional to labor values, they must have the same value composition. Since we previously showed the converse, we can conclude that prices are proportional to labor values if and only if the industries that share the proportionality relationship all have the same value composition of capital. This proves Proposition IV.B.

To prove (V.A), recall from equation (11h) in the proof of (IV.A) that the Marxian price \( q_i \) of any good \( i \) with non-zero variable capital (and hence finite value composition) satisfies:

\[
q_i = (C_i + V_i) + \frac{(k_i+1)S_i}{k+1},
\]

(24)

where \( C_i, V_i, \) and \( S_i \) are the constant capital, variable capital, and surplus value, respectively, of good \( i \); \( k_i = C_i/V_i \) is the value composition of good \( i \), and \( k \) is the social average value composition as defined in equation (8). Since \( \lambda_i = C_i + V_i + S_i \), it follows from (24) that \( q_i > \lambda_i \) if and only if:

\[
(C_i + V_i) + \frac{(k_i+1)S_i}{k+1} = C_i + V_i + S_i
\]

(25a)

\[
\Leftrightarrow \frac{(k_i+1)S_i}{k+1} > S_i
\]

(25b)
\[ \iff (k_i + 1)S_i > (\bar{k} + 1)S_i \quad (25c) \]
\[ \iff k_i + 1 > \bar{k} + 1 \quad (25d) \]
\[ \iff k_i > \bar{k}, \quad (25e) \]

where we have used the fact that \( S_i > 0 \) (since \( V_i > 0 \)) in canceling it from both sides of (25c). Thus we have shown that for a good with non-zero variable capital, its value is lower than its Marxian price if and only if its value composition is higher than the social average. Since the inequalities could just as well have all been flipped, we have also shown that the value of the good is higher than its Marxian price if and only if its value composition is lower than the social average, as stated by Proposition (V.A). A good with zero variable capital has an infinite value composition, which is clearly higher than the social average. But also, its value is \( \lambda_i = C_i \), which is smaller than its Marxian price \( q_i = (1 + \pi)C_i \) in light of \( \pi > 0 \). Thus it is again correct to say that its value is less than its Marxian price if and only if its value composition is higher than the social average. It is also (vacuously) correct to say that its value is higher than its Marxian price if and only if its value composition is lower than the social average, since both statements are always false. Thus, no matter whether the variable capital of a good is positive or zero, its value is lower (higher) than its Marxian price if and only if its value composition is higher (lower) than the social average. This proves Proposition (V.A).

To prove (V.B), assume that linear dependence of industries holds. Then as shown in the proof of (IV.B), for any two goods \( i \) and \( j \) we have:

\[ \frac{p_i}{p_j} = \frac{q_i}{q_j} \quad (26) \]

Also shown in that proof (see equations (15d) and (17c), respectively), for any two goods \( i \) and \( j \) with positive variable capitals we have:

\[ \frac{q_i}{q_j} = \frac{(k_i + 1)V_i}{(k_j + 1)V_j} \quad (27) \]

and

\[ \frac{\lambda_i}{\lambda_j} = \frac{(k_i + 1 + e)V_i}{(k_j + 1 + e)V_j} \quad (28) \]

From (26) and (27) it then follows that:

\[ \frac{p_i}{p_j} = \frac{(k_i + 1)V_i}{(k_j + 1)V_j} \quad (29) \]

if goods \( i \) and \( j \) both have positive variable capitals. Thus, from (28) and (29), \( p_i/p_j > \lambda_i/\lambda_j \) if and only if:

\[ \frac{(k_i + 1)V_i}{(k_j + 1)V_j} > \frac{(k_i + 1 + e)V_i}{(k_j + 1 + e)V_j} \quad (30a) \]
\[
\begin{align*}
\Leftrightarrow \frac{(k_i+1)}{(k_j+1)} & > \frac{(k_i+1+e)}{(k_j+1+e)} \quad (30b) \\
\Leftrightarrow (k_i + 1)(k_j + 1 + e) & > (k_i + 1 + e)(k_j + 1) \quad (30c) \\
\Leftrightarrow (k_i + 1)(k_j + 1) + e(k_i + 1) & > (k_i + 1)(k_j + 1) + e(k_j + 1) \quad (30d) \\
\Leftrightarrow e(k_i + 1) & > e(k_j + 1) \quad (30e) \\
\Leftrightarrow k_i + 1 & > k_j + 1 \quad (30f) \\
\Leftrightarrow k_i & > k_j, \quad (30g)
\end{align*}
\]

where we have used \(V_i/V_j > 0\) in canceling it from both sides of (30a), and \(e > 0\) in canceling it from both sides of (30e).

This shows that for goods with positive variable capitals, the relative price of good \(i\) to good \(j\) is greater than the relative value of \(i\) to \(j\) if and only if the value composition of \(i\) is greater than that of \(j\). Since all of the inequalities could just as well have been flipped, we have also shown that the relative price of \(i\) to \(j\) is less than the relative value of \(i\) to \(j\) if and only if the value composition of \(i\) is less than that of \(j\). If both goods had zero variable capital, then they would both have infinite value composition, so it would not be possible to say that either one has greater value composition than the other. Their relative prices would equal their relative Marxian prices, which would be \(C_i/C_j\). Their relative values would also equal \(C_i/C_j\), so their relative prices and relative values would be equal. Vacuously, then, it is correct to say that the relative price of \(i\) to \(j\) is greater (less) than the relative value of \(i\) to \(j\) if and only if the value composition of \(i\) is greater (less) than that of \(j\), since both of the underlying statements are false. Now suppose one of the goods, say good \(i\), has positive variable capital and the other good \(j\) has zero variable capital. Then good \(j\) has infinite value composition and good \(i\) has finite value composition, so the value composition of \(j\) is certainly greater than that of \(i\). The relative price of \(i\) to \(j\) is equal to the relative Marxian price, which is \((C_i + V_i)/C_j\). The relative value of \(i\) to \(j\) is \((C_i + V_i + S_i)/C_j\), which is greater than the relative price. So it is correct to say that the relative price of \(i\) to \(j\) is less than the relative value of \(i\) to \(j\) if and only if the value composition of \(i\) is less than that of \(j\), since both statements are true. It’s vacuously correct to say that the relative price of \(i\) to \(j\) is greater than the relative value of \(i\) to \(j\) if and only if the value composition of \(i\) is greater than that of \(j\), since both statements are false. Therefore, we have shown that regardless of the configurations of the variable capitals, the relative price of good \(i\) to good \(j\) is greater (less) than the relative value of \(i\) to \(j\) if and only if the value composition of \(i\) is greater (less) than that of \(j\). This proves (V.B).

**DISCUSSION**

Marx’s original version of Proposition (I) stated that the sum of the prices of all goods is equal to the sum of their labor values:¹

“The aggregate price of the commodities I to V would therefore equal their aggregate value, i.e., the sum of the cost-prices I to V plus the sum of the surplus-values, or profits, produced in I to V. It would hence actually be the money-expression of the total quantity of past and newly applied labour incorporated in commodities I to V. And in the same way the sum of the prices of

¹ Quotes from Volume III of *Capital* from: https://www.marxists.org/archive/marx/works/download/pdf/Capital-Volume-III.pdf
production of all commodities produced in society – the totality of all branches of production – is equal to the sum of their values.” - Vol. 3, p. 122

This cannot be correct with regard to the true equilibrium prices because they are measured in different units than labor values (dollars vs. hours). So it would not make sense for them to be equal in the aggregate. It also cannot be correct with regard to the wage-prices because, as shown in Result 9 of “Morishima’s Results for Capitalist Production,” if all industries earn positive profits (as they do under Assumption Blocks 1-6), then the wage-price of each commodity is greater than its labor value. So the sum of the wage-prices must clearly be greater than the sum of the labor values. As Proposition 1 states, however, this result can be salvaged by using the Marxian prices instead of the equilibrium prices or wage-prices and evaluating aggregate price and value at the balanced growth quantities rather than the actual amounts produced.

Marx’s version of Proposition (II) maintained that the cost-price of a good was necessarily less than its value. This is literally true if speaking in terms of labor values. In Volume 3, Marx defines the cost-price in a way that makes it clear that he’s talking about the constant plus variable capital $C_l + V_l$:

“The value of every commodity produced in the capitalist way is represented in the formula: $C = c + v + s$. If we subtract surplus-values from this value of the product there remains a bare equivalent or a substitute value in goods, for the capital-value $c + v$ expended in the elements of production. For example, if the production of a certain article requires a capital outlay of £500, of which £20 are for the wear and tear of instruments of production, £380 for the materials of production, and £100 for labour-power, and if the rate of surplus-value is 100%, then the value of the product = $400c + 100 v + 100s = £600$. After deducting the surplus-value of £100, there remains a commodity-value of £500 which only replaces the expended capital of £500. This portion of the value of the commodity, which replaces the price of the consumed means of production and labour-power, only replaces what the commodity costs the capitalist himself. For him it, therefore, represents the cost-price of the commodity.” - Vol. 3, p. 19

Since the decomposition of value equation states that the labor value of a good is equal to the sum of constant capital, variable capital, and surplus value, it is clearly true that the value exceeds the sum of constant and variable capital (i.e., the cost-price in Marx’s description above) as long as the surplus value is positive. But if the rate of profit is positive, which it is under Assumption Blocks 1-6, then the rate of exploitation must be as well, so that the surplus value will be positive for any good that has positive variable capital. Thus, for any good with positive variable capital the labor value must exceed the cost-price, as observed by Marx in the following passage:

“Therefore, the capitalist is the actual producer of the commodity. For this reason the cost-price of the commodity necessarily appears to the capitalist as the actual cost of the commodity. If we take $k$ to the cost-price, the formula $C = c + v + s$ turns into the formula $C = k + s$, that is, the commodity-value = cost-price + surplus value. The grouping of the various value portions of a commodity which only replace the value of the capital expended in its production under the head of cost-price expresses, on the one hand, the specific character of capitalist production. The capitalist cost of the commodity is measured by the expenditure of capital, while the actual cost
of the commodity is measured by the expenditure of labor. Thus, the capitalist cost-price of the commodity differs in quantity from its value, or its actual cost-price. It is smaller than the value of the commodity, because, with $C = k + s$, it is evident that $k = C – s.$” -Vol. 3, p. 20

However, it’s also clear in Marx’s description that he is using labor hours and monetary units interchangeably when he uses “500 British pounds” to refer to the sum $400c + 100v$, which is the total capital of the good and hence measured in labor hours. If one wants to express the cost-price of the good in monetary terms, it would $C_i^p + V_i^p$ instead of $C_i + V_i$. But then it would not be directly comparable to the labor value. To make it comparable, we could express $C_i^p + V_i^p$ in units of labor by dividing through by the wage $w$. This would give:

$$\frac{1}{w} \{C_i^p + V_i^p\} = \frac{1}{w} \{\sum_{j=1}^{n} p_j a_{ji} + \omega p_{II} B l_i\}$$ (31a)

$$= \frac{1}{w} \{\sum_{j=1}^{n} p_j a_{ji} + w l_i\}$$ (31b)

$$= \{\sum_{j=1}^{n} \frac{p_j}{w} a_{ji} + l_i\}$$ (31c)

$$= \{\sum_{j=1}^{n} p_{j,w} a_{ji} + l_i\}$$ (31d)

where $p_j$ is the true equilibrium price of good $j$, $a_{ji}$ is the amount of capital good $j$ required to produce a unit of good $i$, $p_{II}$ is the vector of equilibrium prices of wage / luxury goods, $l_i$ is the amount of labor required to produce a unit of good $i$, $p_{j,w}$ is the wage-price of good $j$, and we have used the fact that the wage is the cost of the subsistence bundle, i.e., $w = \omega p_{II} B$, in moving from (31a) to (31b). Since the wage-price of each good exceeds its labor value if all industries earn positive profits, and at least one of the $a_{ji}$’s is positive under Assumption Block 2, the expression in (31d) is greater than $\sum_{j=1}^{n} \lambda_j a_{ji} + l_i$, which is simply $\lambda_i$. Thus the cost-price of a good, measured in equilibrium prices and converted to labor units, is strictly greater than, not less than, the labor value of the good. With this conception of the cost-price, Marx’s result is then not true, although what Marx probably meant was that $C_i + V_i < \lambda_i$, which is true as long as good $i$ has positive variable capital. To make Marx’s result true for cost-prices constructed with prices instead of values, we see from Proposition (II) that we should use Marxian prices instead of equilibrium prices, and also impose the assumption of linear dependence of industries.

Marx’s original version of Proposition (III) asserted that the total surplus value earned from the production of all goods equals the aggregate profit on all goods:

“These more definite forms of capitalist production can only be comprehensively presented, however, after the general nature of capital is understood. Furthermore, they do not come within the scope of this work and belong to its eventual continuation. Nevertheless the phenomena listed in the above title may be discussed in a general way at this stage. They are interrelated, first with one another and, secondly, also with the rate and amount of profit. They are to be briefly discussed here if only because they create the impression that not only the rate, but also the amount of profit – which is actually identical with the amount of surplus value – could
increase or decrease independently of the movements of the quantity or rate of surplus-value.” - Vol. 3, p. 79

“We demonstrated in the preceding chapter that, assuming the rate of surplus-value to be constant, the rate of profit obtaining for any given capital may rise or fall in consequence of circumstances which raise or lower the value of one or the other portion of constant capital, and so affect the proportion between the variable and constant components of capital. We further observed that circumstances which prolong or reduce the time of turnover of an individual capital may similarly influence the rate of profit. Since the mass of the profit is identical with the mass of the surplus-value, and with the surplus-value itself, it was also seen that the mass of the profit – as distinct from the rate of profit – is not affected by the aforementioned fluctuations of value.” - Vol. 3, pp. 110-111

“We saw in Part I [of Volume 3] that surplus-value and profit are identical from the standpoint of their mass.” - Vol. 3, p. 126

This is another result that cannot be literally true because of differences in units of measurement: surplus value is measured in labor hours whereas profit is measured in monetary units. But even if we convert profit to labor hours by dividing by the wage, the result still cannot be true. To see this, let \( \Pi_j \) denote profit per unit of output in industry \( j \) (measured in monetary units) and let \( x_j \) denote the number of units of output actually produced in industry \( j \). Since profit per unit is equal to the price of the good minus capital cost per unit (i.e., constant capital in dollars \( C_j^p \) minus labor cost per unit (i.e., variable capital in dollars \( V_j^p \)), we have \( \Pi_j = p_j - C_j^p - V_j^p \). Therefore, total profit summed up across all industries is:

\[
\sum_{j=1}^n \Pi_j x_j = \sum_{j=1}^n \{ p_j - (C_j^p + V_j^p) \} x_j
\]

(32a)

\[
= \sum_{j=1}^n \{ p_j x_j - \sum_{j=1}^n C_j^p x_j - \sum_{j=1}^n V_j^p x_j \}
\]

(32b)

\[
= \sum_{j=1}^n p_j x_j + \sum_{j=n+1}^m p_j x_j - \sum_{j=1}^n C_j^p x_j - \sum_{j=n+1}^m C_j^p x_j - \sum_{j=1}^n V_j^p x_j - \sum_{j=n+1}^m V_j^p x_j
\]

(32c)

\[
= p_l X_l + p_h X_h - [C_1^p \ldots C_n^p] X_l - [C_{n+1}^p \ldots C_m^p] X_h - [V_1^p \ldots V_n^p] X_l
\]

(32d)

\[
= p_l X_l + p_h X_h - \left[ \sum_{j=1}^n p_j a_{j1} \ldots \sum_{j=1}^n p_j a_{jn} \right] X_l - \left[ \sum_{j=1}^n p_j a_{j,n+1} \ldots \sum_{j=1}^n p_j a_{jn+1} \right] X_h
\]

(32e)

\[
- \left[ \omega p_h B l_1 \ldots \omega p_h B l_{n+1} \right] X_l - \left[ \omega p_h B l_{n+1} \ldots \omega p_h B l_m \right] X_l
\]

(32f)

where \( p_l \) is the vector of prices of capital goods, \( X_l \) is the vector of capital goods actually produced, \( X_h \) is the vector of wage and luxury goods produced, \( A_l \) is the capital input coefficient
matrix for capital goods, \( A_{II} \) is the capital input coefficient matrix for wage and luxury goods, \( L_I \) is the labor input coefficient vector for capital goods, \( L_{II} \) is the labor input coefficient vector for wage and luxury goods, \( X_I^* \) is the vector of capital goods that are needed to produce the capital and wage/luxury goods that are actually produced and \( \bar{N} \) is the number of workers actually employed. (See Result 1 in “Morishima’s Results for Capitalist Production” for an explanation of why \( X_I^* = A_I X_I + A_{II} X_{II} \) and \( T \bar{N} = L_I X_I + L_{II} X_{II} \). We used the fact that \( \omega T = 1 \) in moving from (32i) to (32j)).

Note that since \( X_I \) is the vector of capital goods actually produced, and \( X_I^* \) is the vector of necessary amounts of capital goods, the expression \( (X_I - X_I^*) \) in (32k) is the vector of surplus amounts of capital goods. Similarly, \( B \bar{N} \) is the vector of necessary amounts of wage and luxury goods because \( B \) is the vector of subsistence amounts per worker and \( \bar{N} \) is the number of workers employed. Since \( X_{II} \) is the vector of wage and luxury goods actually produced, it follows that the expression \( X_{II} - B \bar{N} \) in (32k) is the vector of surplus amounts of wage and luxury goods. Hence the entire expression in (32k) gives the aggregate value of surplus quantities of all goods valued in market prices, i.e., the aggregate surplus value in market prices. Thus we see in the above derivation that total profit across all industries equals aggregate surplus value in market prices.

It follows then that total profit measured in labor hours is equal to the aggregate value of surplus output measured in wage-prices:

\[
\frac{1}{w} \sum_{j=1}^{m} \Pi_j x_j = \frac{p_{I}}{w} (X_I - X_I^*) + \frac{p_{II}}{w} (X_{II} - B \bar{N}) = p_{I,w} (X_I - X_I^*) + p_{II,w} (X_{II} - B \bar{N}),
\]

(33a) \hfill (33b)

where \( p_{I,w} \) is the vector of wage-prices of capital goods and \( p_{II,w} \) is the vector of wage-prices of wage and luxury goods. Now since the rate of profit \( \pi \) is profit per unit \( \Pi_j \) divided by production cost \( p_I \text{col}_j A_I + w l_j \) for any good \( j \) (see equations (3a) and (3b) in “Existence of Capitalist Equilibrium” and impose a common rate of profit \( \pi \) in place of \( \pi_j \)), the production cost is strictly positive since \( p_I > 0 \) under Assumption Blocks 1-5 and \( \text{col}_j A_I \) has at least one strictly positive element under Assumption Block 2, and the rate of profit is strictly positive under Assumption Blocks 1-6, it follows that \( \Pi_j > 0 \) for all \( j \). Since the wage is also strictly positive (it’s equal to \( \omega p_{II} B \) with \( \omega > 0 \), \( p_{II} > 0 \) and \( B \) having at least one strictly positive element) and at least one of the \( x_j \)’s will be strictly positive considering that these are the amounts produced, it follows that \( w^{-1} \sum_{j=1}^{m} \Pi_j x_j \) is strictly positive. But then the expression in (33b) is strictly positive as well. Since \( p_{I,w} > 0 \) and \( p_{II,w} > 0 \) under Assumption Blocks 1-5 and the fact that \( w > 0 \), while \( (X_I - X_I^*) \) and \( (X_{II} - B \bar{N}) \) must both be \( \geq 0 \) since at least the socially necessary amounts of each good must be produced in equilibrium,\(^2\) it must be true that either \( (X_I - X_I^*) \) or \( (X_{II} - B \bar{N}) \) has at least one strictly positive entry. But the fact that wage-prices are strictly greater than labor values when all industries earn positive profits implies \( p_{I,w} > \Lambda_I \) and \( p_{II,w} > \Lambda_{II} \), which then implies:

\[
p_{I,w} (X_I - X_I^*) + p_{II,w} (X_{II} - B \bar{N}) > \Lambda_I (X_I - X_I^*) + \Lambda_{II} (X_{II} - B \bar{N})
\]

(34)

\(^2\) A set of vectors \( X_I \) and \( X_{II} \) that satisfies these conditions definitely exists. See the Appendix for a proof.
The right-hand side of (34) is the aggregate labor value of the surplus output. This is actually equal to the aggregate surplus value, since:

\[ \Lambda_1(X_i - X_i^*) + \Lambda_{II}(X_{II} - B\bar{N}) = \Lambda_1X_i + \Lambda_{II}X_{II} - \Lambda_1X_i^* - \Lambda_{II}B\bar{N} \]  
\[ = \Lambda_1X_i + \Lambda_{II}X_{II} - \Lambda_1(A_1X_i + A_{II}X_{II}) - \omega\Lambda_{II}BT\bar{N} \]  
\[ = \Lambda_1X_i + \Lambda_{II}X_{II} - \Lambda_1A_1X_i - \Lambda_1A_{II}X_{II} - \omega\Lambda_{II}B(L_1X_i + L_{II}X_{II}) \]  
\[ = \Lambda_1X_i + \Lambda_{II}X_{II} - \left[ C_1 \cdots C_n \right]X_i - \left[ C_{n+1} \cdots C_m \right]X_{II} - \omega\Lambda_{II}BL_1X_i - \omega\Lambda_{II}BL_{II}X_{II} \]  
\[ = \sum_{j=1}^{n} \lambda_jx_j + \sum_{j=n+1}^{m} \lambda_jx_j - \sum_{j=1}^{n} \lambda_jx_j - \sum_{j=n+1}^{m} \lambda_jx_j - \sum_{j=n+1}^{n} V_jx_j - \sum_{j=n+1}^{m} V_jx_j \]  
\[ = \sum_{j=1}^{n} \lambda_jx_j - \sum_{j=1}^{n} \lambda_jx_j - \sum_{j=n+1}^{m} \lambda_jx_j - \sum_{j=n+1}^{m} \lambda_jx_j \]  
\[ = \sum_{j=1}^{n} \lambda_jx_j - \sum_{j=n+1}^{m} \lambda_jx_j \]  
\[ = \sum_{j=1}^{m} (\lambda_j - C_j - V_j)x_j \]  
\[ = \sum_{j=1}^{m} S_jx_j, \]  
where we have used \( X_i^* = A_1X_i + A_{II}X_{II} \) and \( \omega T = 1 \) in moving from (35a) to (35b), \( T\bar{N} = L_1X_i + L_{II}X_{II} \) in moving (35b) to (35c), the fact that \( \Lambda_1A_1 \) and \( \Lambda_1A_{II} \) are the vectors of constant capitals of capital goods and wage/luxury goods, respectively, in moving from (35c) to (35d); the fact that \( \omega\Lambda_{II}BL_1 \) and \( \omega\Lambda_{II}BL_{II} \) are the vectors of variable capitals of capital goods and wage/luxury goods, respectively, in moving from (35d) to (35e); and the decomposition of value equation in moving from (35h) to (35i).

Putting all of our results in this section together then shows that total profits measured in labor units are strictly greater than, not equal to, total surplus value:

\[ \frac{1}{w} \sum_{j=1}^{m} \Pi_jx_j = p_{I,w}(X_i - X_i^*) + p_{II,w}(X_{II} - B\bar{N}) \]  
\[ > \Lambda_1(X_i - X_i^*) + \Lambda_{II}(X_{II} - B\bar{N}) \]  
\[ = \sum_{j=1}^{m} S_jx_j \]  
As Proposition (III) shows, making the result correct requires us to assume linear dependence of industries and to evaluate profits and surplus values at the balanced growth vector instead of the vector of goods actually produced.

The original Proposition (IV) stated that the price of production of commodities equal their value only in spheres in which the value composition of capital is the same as the social average, as we see in the following quote, which also contains the original Proposition (V); see below:

“Such capitals as contain a larger percentage of constant and a smaller percentage of variable capital than the average social capital are, therefore, called capitals of higher composition, and, conversely, those capitals in which the constant is relatively smaller, and the variable relatively greater than in the averages social capital, are called capitals of lower composition. Finally, we call those capitals whose composition coincides with the average, capitals of average composition. Should the average social capital be composed in per cent of 80c + 20v, then a capital of 90c + 10v, is higher, and a capital of 70c + 30v, lower than the social average…The way in which these capitals perform their functions after establishment of an average rate of profit
and assuming one turnover per year, is shown in the following tabulation, in which I represents the average composition with an average rate of profit of 20%.

I- 80c + 20v + 20s. Rate of profit = 20%
   Price of product = 120. Value = 120.
II-90c + 10v + 10s. Rate of profit = 20%
   Price of product = 120. Value = 110.
III-70c + 30v + 30s. Rate of profit = 20%
   Price of product = 120. Value = 130.

The value of the commodities produced by capital II would, therefore, be smaller than their price of production, the price of production of the commodities of III smaller than their value, and only in the case of capital I in branches of production in which the composition happens to coincide with the social average, would value and price of production be equal.” -Vol. 3, pp. 124-125

The assertion “price equals value when value composition equals the social average” cannot be true of the equilibrium prices since they are measured in monetary units rather than labor hours. It also cannot be true of the wage-prices since those are all strictly greater than labor values when there are positive profits earned in all industries. It could potentially be true of Marxian prices since those are measured in labor values. Indeed, version (A) of the rehabilitated proposition indicates that it is true of the Marxian prices in balanced growth equilibrium. Version (B) shows that another way to make the result correct is to assume linear dependence of industries and then adjust for the difference in units between prices and labor values by allowing for them to be proportional rather than insisting they be equal. But in this case the proportionality holds among all goods with the same value composition of capital, whether or not it coincides with the social average.

The original Proposition (V) stated that the value of commodities produced by capital of higher (lower) value composition than the average is smaller (larger) than their price of production, and we see it embedded in the above quote. This cannot be true of the true equilibrium prices due to the mismatch in units with labor values. It also cannot be true of the wage-prices since there are positive profits earned in all industries. The result is true, however, if stated in terms of the Marxian prices of production rather than the true equilibrium prices (version A). It can also be rehabilitated by assuming linear dependence of industries and making a claim about relative prices versus relative values based on which good has the higher value composition (version B).

**APPENDIX: THERE EXIST VECTORS X_I AND X_{II} OF CAPITAL GOODS AND WAGE / LUXURY GOODS, RESPECTIVELY, THAT SATISFY THE CONDITION THAT THE ACTUAL AMOUNTS PRODUCED ARE AT LEAST AS LARGE AS THE SOCIALLY NECESSARY AMOUNTS, I.E. X_I \geq X_I^* and X_{II} \geq B \bar{N}.

Since by definition X_I^* = A_I X_I + A_{II} X_{II} and \bar{N} = \omega L_I X_I + \omega L_{II} X_{II}, where \omega = 1/T, observe that the conditions X_I \geq X_I^* and X_{II} \geq B \bar{N} are respectively equivalent to:
\[ X_I \geq A_I X_I + A_{II} X_{II} \]  
\[ X_{II} \geq \omega B L_I X_I + \omega B L_{II} X_{II} \]  

These conditions are equivalent to:

\[ (I - A_I) X_I - A_{II} X_{II} \geq 0 \]  
\[ -\omega B L_I X_I + (I - \omega B L_{II}) X_{II} \geq 0 \]

Putting these inequalities in matrix form yields:

\[
\begin{bmatrix}
(I - A_I) & -A_{II} \\
-\omega B L_I & (I - \omega B L_{II})
\end{bmatrix}
\begin{bmatrix}
X_I \\
X_{II}
\end{bmatrix} \geq 0
\]  
\( \Leftrightarrow \left\{ \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} A_I & A_{II} \\ \omega B L_I & \omega B L_{II} \end{bmatrix} \right\} \begin{bmatrix} X_I \\
X_{II} \end{bmatrix} \geq 0 \)

where \( M \) is the input power matrix and \( X \) is the column vector with \( X_I \) as its first \( n \) entries and \( X_{II} \) as its next \((m-n)\) entries. Recall from Lemma 1 that under Assumption Blocks 1, 5, and 6, \( M \) is productive. By definition, this means there exists a vector \( X > 0 \) such that \((I - M)X > 0\). So (39c) would hold with strict inequality as would (37a) and (37b). What we have literally shown then is that there is a vector of strictly positive production amounts of all goods such that the amount of each good produced is strictly greater than the socially necessary amount.