RESULT 3: UNDER ASSUMPTION BLOCKS 1-6, ALL INDUSTRIES HAVE THE SAME VALUE COMPOSITION OF CAPITAL IF AND ONLY IF THE VECTOR FORM OF THE MORISHIMA-SETON EQUATION HOLDS: $S = \pi(C+V)$, WHERE $\pi$ IS THE RATE OF PROFIT AND $S$, $C$, AND $V$ ARE THE VECTORS OF SURPLUS VALUES, CONSTANT CAPITALS, AND VARIABLE CAPITALS, RESPECTIVELY, OF ALL GOODS.

The reason that the equation $S = \pi(C+V)$ is called the “vector form” of the Morishima-Seton equation is that the actual Morishima-Seton equation, which we could refer to as the “scalar form” of the equation, says that $\pi = S/(C+V)$, where $C$, $V$, and $S$ are the sum of the constant capitals, variable capitals, and surplus values, respectively, of the quantities of the goods in the characteristic vector $y$. Therefore $S = \pi(C+V)$, where $C$, $V$, and $S$ are numbers rather than vectors. The result here simply asserts that under the assumptions given we can replace these numbers with vectors (i.e., the vectors of constant capitals, variable capitals, and surplus values of all goods) and it will still be true.

We need to prove two implications:

All industries have the same value composition of capital $\Rightarrow S = \pi(C+V)$

and

$S = \pi(C+V) \Rightarrow$ all industries have the same value composition of capital

To prove the first implication, assume that all industries have the same value composition of capital $k$. Since $C_i > 0$ for all $i$ under our assumptions (labor values are all strictly positive under Assumption Blocks 1-3, and every column of $A_i$ and of $A_{ii}$ has at least one strictly positive entry under Assumption Block 2), the only two possibilities are that $k$ is a finite positive real number or that $k$ is infinite. If $k$ is infinite, it means that $V_i = 0$ for all $i$. Then $S_i = eV_i = 0$ for all $i$, and the Morishima-Seton equation, which holds under Assumption Blocks 1-5, gives:

$$\pi = \frac{\sum_{j=1}^{m} s_i y_j}{\sum_{j=1}^{m} C_j y_j + V_j y_j},$$

$$= \frac{\sum_{j=1}^{m} C_j y_j}{\sum_{j=1}^{m} C_j y_j}$$

$$= 0,$$

since $\sum_{j=1}^{m} C_j y_j > 0$ in view of $C_j > 0$ for all $j$ and the characteristic vector $y$ being a non-negative, non-zero vector. Thus, we have $S = 0$ and $\pi(C+V) = 0$, so that the result $S = \pi(C+V)$ holds. We may therefore assume from here on out that $k$ is a finite positive real number, which implies that $V_i > 0$ for all $i$. Note that since $S_i = eV_i$ for all $i$, we have $S = eV$, and since $C_i/V_i = k$ for all $i$, we have $C = kV$. Furthermore, the Morishima-Seton equation (1a) yields:
\[\pi = \frac{e^{\sum_{j=1}^{m} y_j}}{k^{\sum_{j=1}^{m} y_j + y_j y_j}}\]  
\[= \frac{e^{y_j \beta_j / \sum_{j=1}^{m} y_j \beta_j}}{k^{1 + \sum_{j=1}^{m} y_j / \sum_{j=1}^{m} y_j \beta_j}}\]  
\[= \frac{e}{k^{1 + \sum_{j=1}^{m} y_j / \sum_{j=1}^{m} y_j \beta_j}}\]  
\[= \frac{e}{k^{1 + y_j / \beta_j}}\]  
\[= \frac{e}{k^{1 + y_j / \beta_j}}\]  

where we have used the fact that \(\sum_{j=1}^{m} y_j \beta_j > 0\) in light of \(y\) being a non-negative, non-zero vector and \(y_j > 0\) for all \(j\). It follows from (2c) that \(e = \pi(k+1)\), so that:

\[S = eV\]  
\[= \pi(k + 1)V\]  
\[= \pi(kV + V)\]  
\[= \pi(C + V),\]  

so the vector form of the Morishima-Seton equation holds and the first implication is proved.

To prove the second implication, assume that \(S = \pi(C + V)\) holds as a vector equation. Then since \(S = eV\), we have:

\[eV = \pi(C + V)\]  
\[= \pi C + \pi V,\]  
\[\Rightarrow (e - \pi)V = \pi C\]  
\[\Rightarrow C = \frac{(e - \pi)}{\pi} V\]  
\[\Rightarrow C = kV,\]  

where \(k = (e - \pi) / \pi\) and we have used the fact that \(\pi > 0\) under Assumption Blocks 1-6. Also, since \(\pi > 0\), Corollary 1 to the Morishima-Seton equation implies that \(\pi < e\), so that \(k > 0\). By (4e) we then have \(C = kV\) with \(k\) a finite positive real number. Hence \(C_i = kV_i\) for all \(i\), and none of the \(V_i\) can be zero because then \(C_i\) would be zero, which is not possible under our assumption blocks. We then have \(C/V_i = k\) for all \(i\), so all goods have the same (finite positive) value composition of capital. This proves the second implication, and so the proof is now complete.