RESULT 2: UNDER ASSUMPTION BLOCKS 1-6, THE MARXIAN PRICES EQUAL LABOR VALUES IF AND ONLY IF ALL INDUSTRIES HAVE THE SAME VALUE COMPOSITION OF CAPITAL.

The “Marxian prices” are calculated as a markup over the total capital, where the markup factor is 1 plus the rate of profit. So, for example, if the rate of profit is 8%, then the Marxian price of good \( i \) would be 1.08 times the sum of the constant and variable capital of good \( i \). Thus the Marxian price for good \( i \), which we denote \( q_i \), satisfies the following for all \( i=1,\ldots,m \):

\[
q_i = (1 + \pi)(C_i + V_i),
\]

(1)

where \( C_i \) is the constant capital of good \( i \), \( V_i \) is the variable capital of good \( i \), and \( \pi \) is the rate of profit. Recall that \( C_i = \sum_{j=1}^{n} \lambda_j a_{ij} \) and \( V_i = \omega \Lambda_H B l_i \), where \( \lambda_j \) is the labor value of good \( j \), \( a_{ij} \) is the number of units of capital good \( j \) needed to produce a unit of good \( i \), \( l_i \) is the number of units of labor needed to produce a unit of good \( i \), \( \omega = 1/T \) is the fraction of the workday \( T \) taken up by an hour, \( \Lambda_H \) is the vector of labor values of wage and luxury goods, and \( B \) is the vector of subsistence quantities of wage and luxury goods. Under Assumption Blocks 1-3, \( C_i > 0 \) and \( V_i \geq 0 \) for all \( i \), with \( V_i > 0 \) for at least one capital good and at least one wage/luxury good.

Note that the Marxian prices as defined by (1) are measured in the same units as constant and variable capital, i.e., units of labor. As such, they can (unlike the actual market prices) be directly compared to labor values. Indeed, this definition appears to match up well with how Marx defined the “prices of production” as, for example, in this passage from Volume III:

“The formula that the price of production of a commodity = k + p, i.e., equals its cost-price plus profit, is now more precisely defined with p = kp’ (p’ being the general rate of profit). Hence the price of production = k + kp’.” Vol. 3, p. 125

To unpack this definition further, observe that the cost-price is defined as follows in Volume III:

“The value of every commodity produced in the capitalist way is represented in the formula: \( C = c + v + s \). If we subtract surplus-values from this value of the product there remains a bare equivalent or a substitute value in goods, for the capital-value \( c + v \) expended in the elements of production. For example, if the production of a certain article requires a capital outlay of £500, of which £20 are for the wear and tear of instruments of production, £380 for the materials of production, and £100 for labour-power, and if the rate of surplus-value is 100%, then the value of the product = \( 400c + 100v + 100s = £600 \). After deducting the surplus-value of £100, there remains a commodity-value of £500 which only replaces the expended capital of £500. This portion of the value of the commodity, which replaces the price of the consumed means of production and labour-power, only replaces what the commodity costs the capitalist himself. For him it, therefore, represents the cost-price of the commodity.” -Vol. 3, p. 19

\( ^{1} \) Quotes from Volume III of Capital from: https://www.marxists.org/archive/marx/works/download/pdf/Capital-Volume-III.pdf
It is clear from this passage that the cost-price refers to the term \(c+v\), which is the sum of the constant and the variable capital. Putting this together with the formula for price from the quote from p. 125, we see that Marx intends for the price of production to be expressed, using our notation, as \((1+p')(C_i + V_i)\), where \(p'\) is the rate of profit. Thus, Morishima’s notion of the “Marxian price” is in fact an exact replica of Marx’s own formula for the price of production.

The motivation for defining prices this way, other than to match up with Marx’s own definition, is that a number of results posited by Marx (e.g., sum of prices equals sum of labor values) are not literally true for market prices as we normally think of them, but are true for prices as defined in (1). Thus, some of Marx’s key results were not true for the actual market prices of goods, but did turn out to be true for prices as he defined them.

Recall from Existence of Capitalist Equilibrium that under Assumption Blocks 1, 4, and 5, we can identify a unique rate of profit \(\pi \geq 0\) and price vector \(p > 0\) such that the price-determining equation \(p = (1 + \pi)pM\) is satisfied, where \(M\) is the input power matrix and the wage is the cost of the hourly subsistence bundle of goods (\(\omega p_H B\)). Corollary 2 to the Morishima-Seton Equation then implies that under Assumption Blocks 1-6, the equilibrium rate of profit satisfies:

\[
\pi = \frac{S}{C+v}, \tag{2}
\]

where \(C\) is the constant capital, \(V\) is the variable capital, and \(S\) is the surplus value of the characteristic vector \(y\), i.e., \(C = \sum_{j=1}^{m} C_j y_j\), \(V = \sum_{j=1}^{m} V_j y_j\), and \(S = \sum_{j=1}^{m} S_j y_j\) where \(y_j\) is the \(j\)th entry of \(y\). Plugging these expressions for \(C, V,\) and \(S\) into (2) yields:

\[
\pi = \frac{\sum_{j=1}^{m} S_j y_j}{\sum_{j=1}^{m} C_j y_j + \sum_{j=1}^{m} V_j y_j} \tag{3}
\]

We are now ready to proceed with the proof. To establish the “if and only if” result, we need to prove two implications:

**All industries have the same value composition of capital \(\Rightarrow\) the Marxian price of each good is equal to its labor value**

and

**The Marxian price of each good is equal to its labor value \(\Rightarrow\) all industries have the same value composition of capital**

To prove the first implication, assume all industries have the same value composition of capital and denote this common value by \(k\), so that \(k = C_i/V_i\) for all \(i=1, \ldots, m\). Suppose that \(V_i = 0\) for some \(i\). Then since \(C_i > 0\) for all \(i\), \(C_i/V_i = \infty\) for this \(i\), so it has to be infinite for all \(i\) since it is the same for all. Hence \(V_i = 0\) for all \(i\), which implies \(S_i = eV_i = 0\) for all \(i\), so that \(\pi = 0\) by (3). But then the Marxian price of good \(i\) is \(q_i = C_i\) by (1). Meanwhile, the labor value is \(\lambda_i = C_i + V_i + S_i = C_i + V_i + eV_i = C_i\), which is in agreement with the Marxian price, so the result holds. Thus we may proceed from here under the assumption \(V_i > 0\) for all \(i\), which implies that
the common value composition of capital $k$ is a finite non-negative real number. Then, as shown in Result 1 (see Equation (20d) in that document), the labor value of good $i$ satisfies:

$$\lambda_i = (1 + \hat{r})(C_i + V_i) \tag{4a}$$

for all $i = 1, \ldots, m$, where

$$\hat{r} = \frac{e}{k+1}. \tag{4b}$$

But observe that by substituting $S_i = eV_i$ and $C_i = kV_i$ into (3), the rate of profit satisfies:

$$\pi = \frac{\sum_{j=1}^{m} eV_j y_j}{\sum_{j=1}^{m} kV_j y_j + \sum_{j=1}^{m} V_j y_j} \tag{5a}$$

$$= \frac{\sum_{j=1}^{m} eV_j y_j}{k \sum_{j=1}^{m} V_j y_j + \sum_{j=1}^{m} V_j y_j} \tag{5b}$$

$$= \frac{e \sum_{j=1}^{m} V_j y_j}{(k+1) \sum_{j=1}^{m} V_j y_j} \tag{5c}$$

$$= \frac{e}{(k+1)} \tag{5d}$$

$$= \hat{r}, \tag{5e}$$

where (5e) follows from the fact that $\sum_{j=1}^{m} V_j y_j > 0$ must hold, since $y$ is a non-zero vector and each $V_j$ is strictly positive by assumption. Substituting (5f) into (4a) and using (1), we obtain:

$$\lambda_i = (1 + \pi)(C_i + V_i) \tag{6a}$$

$$= q_i. \tag{6b}$$

so that the Marxian price of each good is equal to its labor value, proving the first implication.

To prove the second implication, assume that the Marxian price of each good $q_i$ is equal to its labor value $\lambda_i$. Then since $\lambda_i = C_i + V_i + S_i$ by the decomposition of labor value, equation (1) implies that for all $i = 1, \ldots, m$, we have:

$$q_i = \lambda_i = C_i + V_i + S_i \tag{7a}$$

$$\Rightarrow (1 + \pi)(C_i + V_i) = C_i + V_i + S_i \tag{7b}$$

$$\Rightarrow (C_i + V_i) + \pi(C_i + V_i) = (C_i + V_i) + S_i \tag{7c}$$

$$\Rightarrow \pi(C_i + V_i) = S_i \tag{7d}$$

$$\Rightarrow \pi(C_i + V_i) = eV_i \tag{7e}$$

Note that each $V_i$ must be $> 0$, for if $V_i = 0$ for some $i$, then (7e) implies that $\pi(C_i) = 0$, so that $C_i = 0$ since $\pi > 0$ under Assumption Blocks 1, 4, 5, and 6 (under 1, 4, and 5, the Existence of Capitalist Equilibrium tells us $\pi \geq 0$; Assumption Block 6 then implies $\pi > 0$). But this contradicts the fact that $C_i > 0$ for all $i$ under Assumption Blocks 1-3. Thus $V_i > 0$ for all $i$, and we can divide (7e) through by $V_i$ to obtain:
\[ \pi \left( \frac{c_i}{v_i} + 1 \right) = e \]  

(8)

Since \( \pi > 0 \), we can divide (8) through by \( \pi \) and subtract 1 from both sides to obtain:

\[ \frac{c_i}{v_i} = \frac{e}{\pi} - 1 \]  

(9)

Since neither \( e \) nor \( \pi \) depends on \( i \), equation (9) shows that the value composition of capital is the same for all goods. This proves the second proposition, and now the proof is complete.

**DISCUSSION**

This result confirms that the following assertion of Marx is valid for the Marxian prices:

“The capital invested in some spheres of production has a mean, or average, composition, that is, it has the same, or almost the same composition as the average social capital. In these spheres the price of production is exactly or almost the same as the value of the produced commodity expressed in money….In the case of capitals of average, or approximately average, composition, the price of production is thus the same or almost the same as the value, and the profit the same as the surplus-value produced by them.” - Vol. 3, p. 130

However, the result is not valid for the actual market prices. In fact, it cannot be valid for those because they are not even expressed in the same units as labor values. As we showed in the results on Exploitation Theory, it cannot even be valid for the wage-prices, which are market prices converted into labor units by dividing by the wage, as these all must be bigger than the corresponding labor values if all industries are to earn positive profits. But, as we have shown here, Marx did have good intuition in that he realized a certain conception of prices would equal labor values in spheres with a particular common value of the value composition of capital. As such, Morishima’s result can be considered a “rehabilitation” of Marx’s original assertion. It turns out that a number of Marx’s other results can be rehabilitated in a similar fashion.