EXISTENCE OF CAPITALIST EQUILIBRIUM: UNDER ASSUMPTION BLOCKS 1, 4, AND 5, A UNIQUE SUBSISTENCE-WAGE EQUILIBRIUM EXISTS IN THE CAPITALIST ECONOMY. THAT IS, IT IS POSSIBLE TO IDENTIFY A UNIQUE NON-NEGATIVE RATE OF PROFIT $\pi$ AND A UNIQUE STRICTLY POSITIVE PRICE VECTOR $p$ SUCH THAT THE PRICE-DETERMINING EQUATIONS $p = (1+\pi)pM$ ARE SATISFIED WITH THE WAGE EQUAL TO THE COST OF THE HOURLY SUBSISTENCE BUNDLE OF GOODS $\omega B$.

{First recall the relevant assumption blocks:

Assumption Block 1: The matrices $A_I$ and $A_{II}$ and the vectors $L_I$ and $L_{II}$ are all non-negative.
Assumption Block 4: The vector $B$ of subsistence quantities of wage and luxury goods is nonnegative and nonzero; the lower bound for the length of the workday $T$ is the amount of labor time $\Lambda_I B$ required to produce this vector.
Assumption Block 5: The input power matrix $M$ is indecomposable and has a spectral radius no greater than 1.}

Before proving this result, we need some clarification on what is meant by the “rate of profit” and why it should be regarded as unique in a capitalist equilibrium. In accordance with standard definitions in economics, the profit per unit of output for industry $i$ is the price of good $i$ minus the capital and labor cost of production per unit of output of good $i$:

$$\text{profit}_i = p_i - (p_1 a_{1i} + p_2 a_{2i} + \cdots + p_n a_{ni}) - w l_i$$

$$= p_i - \begin{bmatrix} a_{1i} & \cdots & a_{ni} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} - w l_i,$$

(1a)

(1b)

where $p_i$ is the price of good $i$, $p_j$ is the price of capital good $j$ ($j=1,\ldots,n$), $a_{ji}$ is the number of units of capital good $j$ needed to produce a unit of good $i$, $w$ is the wage, and $l_i$ is the number of units of labor needed to produce a unit of good $i$. For capital goods and for wage/luxury goods, equation (1b) can be respectively rewritten as:

$$\text{profit}_i = p_i - p_j c o l_i A_I - w l_i \quad \text{for } i=1,\ldots,n$$

$$\text{profit}_i = p_i - p_j c o l_{i-n} A_{II} - w l_i \quad \text{for } i=n+1,\ldots,m,$$

(2a)

(2b)

where $A_I$ is the capital input matrix for capital goods and $A_{II}$ is the capital input matrix for wage and luxury goods. By the “rate of profit” for industry $i$, which we denote $\pi_i$, we mean its profit per unit relative to its per-unit costs of production. Thus for capital goods and for wage / luxury goods, we respectively have rates of profit given by:

$$\pi_i = \frac{p_i - p_j c o l_i A_I - w l_i}{p_j c o l_i A_I + w l_i} \quad \text{for } i=1,\ldots,n$$

$$\pi_i = \frac{p_i - p_j c o l_{i-n} A_{II} - w l_i}{p_j c o l_{i-n} A_{II} + w l_i} \quad \text{for } i=n+1,\ldots,m$$

(3a)

(3b)
Marx actually defines the rate of profit as follows in Volume III:¹

“The surplus-value, or profit, consists precisely in the excess value of a commodity over its cost-price, i.e., the excess of the total labour embodied in the commodity over the paid labour embodied in it. The surplus-value, whatever its origin, is thus a surplus over the advanced total capital. The proportion of this surplus to the total capital is therefore expressed by the fraction s/C, in which C stands for total capital. We thus obtain the ‘rate of profits’/C = s/(c+v), as distinct from the rate of surplus-value s/v. The rate of surplus-value measured against the variable capital is called the rate of surplus-value. The rate of surplus-value measured against the total capital is called rate of profit.” -Vol. 3, pp. 28-29

Surplus value (s), constant capital (c) and variable capital (v) are all measured in units of labor time. So Marx’s rate of profit is the ratio of two measurements of labor time. The rate of profit as we have defined it here is the ratio of the amount of profit in dollars to the cost of production in dollars; it is the ratio of two sums of money. This is more like the modern definition of profit, which is also in terms of money. Today’s microeconomists will speak of profit as a total but not so much the rate of profit. Our definition therefore blends modern and classical elements. The key thing to notice though is that our definition of the rate of profit is parallel to Marx’s, in that the numerator, or the dollar amount of profit earned, is supposed to be analogous to the surplus value. The denominator, or the dollar amount of production cost, is analogous to the total capital, or the sum of the constant and variable capital. Marx seems to equate these things by speaking of surplus value and total capital in monetary-sounding terms when he says:

“Surplus-value itself does not appear as the product of the appropriation of labour-time, but as an excess of the selling price of commodities over their cost-price, the latter thus being easily represented as their actual value (valeur-intrinseque), while profit appears as an excess of the selling price of commodities over their immanent value.” Vol. 3, p. 29

If we regard both “cost-price” and “immanent value” as the “cost of production in dollar terms”, then this statement of Marx’s would seem to support labeling price minus the cost of production as profit and profit divided by cost of production as the rate of profit. Yet despite the similarities, it still remains a task for us to establish if, and under what conditions, our definition of the rate of profit is really equivalent to Marx’s literal definition.

The justification for focusing on the rate of profit as defined in (3a) and (3b) or as in Marx’s literal definition is found in the following statement in Volume III:

“So far as the individual capitalist is concerned, it is evident that he is only interested in the relation of the surplus-value, or the excess value at which he sells his commodities, to the total capital advanced for the production of the commodities, while the specific relationship and inner connection of this surplus with the various components of capital fail to interest him, and it is, moreover, rather in his interests to draw the veil over this specific relationship and this intrinsic connection.” -Vol. 3, p. 29

¹ Quotes from Capital Volume III taken from: https://www.marxists.org/archive/marx/works/download/pdf/Capital-Volume-III.pdf
Thus in Marx’s view, the item of primary interest to the capitalist is not the numerical size of profit but its size in relation to the total capital advanced (sum of constant and variable capital in Marx’s definition, or the dollar value of the cost of production in ours).

Now rearranging and solving (3a) for \( p_i \), we get:

\[
\pi_i \left( p_i c_i + w_i \right) = p_i - p_i c_i A_i - w_i
\]

\[
\Rightarrow p_i = \pi_i \left( p_i c_i + w_i \right) + (p_i c_i A_i + w_i)
\]

\[
\Rightarrow p_i = \left( 1 + \pi_i \right) \left( p_i c_i A_i + w_i \right)
\]

for \( i = 1, \ldots, n \). Doing the same with (3b) yields:

\[
\pi_i \left( p_i c_i A_i + w_i \right) = p_i - p_i c_i A_i - w_i
\]

\[
\Rightarrow p_i = \pi_i \left( p_i c_i A_i + w_i \right) + (p_i c_i A_i + w_i)
\]

\[
\Rightarrow p_i = \left( 1 + \pi_i \right) \left( p_i c_i A_i + w_i \right)
\]

for \( i = n+1, \ldots, m \).

A standard assumption made in these Marxian models is that the rate of profit is the same across all industries, so that we can write \( \pi_i = \pi \) for \( i = 1, \ldots, m \). The idea behind this assumption is that if the rate of profit were higher in some industries than others, entrepreneurs would move their capital out of industries with lower rates of profit and into industries with higher rates. This process would continue until all rates of profit were equalized. Thus these models contain an implicit assumption of a very high degree of capital mobility across industries. Technically, it is an assumption of perfect capital mobility. Marx believed rates of profit would generally differ:

“Assuming a uniform degree of exploitation, we have seen that regardless of all modifications originating in the credit system, regardless of the capitalists’ efforts to outwit and cheat one another, and, lastly, regardless of any favourable choice of the market – the rate of profit may differ considerably, depending on the low or high prices of raw materials and the experience of the buyer, on the relative productivity, efficiency and cheapness of the machinery, on the greater or lesser efficiency of the aggregate arrangement of the various stages of the productive process, elimination of waste, the simplicity and efficiency of management and supervision, etc. In short, given the surplus-value for a certain variable capital, it still depends very much on the individual business acumen of the capitalist, or of his managers and salesmen, whether this same surplus-value is expressed in a greater or smaller rate of profit, and accordingly yields a greater or smaller amount of profit.” -Vol. 3, p. 108

“This difference in the presentation of the same mass of surplus-value, or the difference in the rates of profit, and therefore in the profit itself, while the exploitation of labour is the same, may also be due to other causes. Still, it may also be due wholly to a difference in the business acumen with which both establishments are run. And this circumstance misleads the capitalist, convinces him that his profits are not due to exploiting labour, but, at least in part, to other independent circumstances, and particularly his individual ability.” -Vol. 3, p. 108

Essentially, Marx saw differing rates of profits across businesses or industries despite similar rates of exploitation as possible due to market conditions, aspects of the production process and
equipment, and the business aptitude of capitalists and managers. However, consistent with our approach here, he felt that the rates of profit would eventually converge due to competition:

“Accordingly, the rates of profit prevailing in the various branches of production are originally very different. These different rates of profit are equalized by competition to a single general rate of profit, which is the average of all these different rates of profit. The profit accruing in accordance with this general rate of profit to any capital of a given magnitude, whatever its organic composition, is called the average profit.” -Vol. 3, p. 121

“What competition, first in a single sphere, achieves is a single market-value and market-price derived from the various individual values of commodities. And it is competition of capitals in different spheres, which first brings out the price of production equalizing the rates of profit in the different spheres. The latter process requires a higher development of capitalist production than the previous one.” -Vol. 3, p. 134

“But capital withdraws from a sphere with a low rate of profit and invades others, which yield a higher profit. Through this incessant outflow and influx, or, briefly, through its distribution among the various spheres, which depends on how the rate of profit falls here and rises there, it creates such a ratio of supply to demand that the average profit in the various spheres of production becomes the same, and values are, therefore, converted into prices of production.” -Vol. 3, p. 142

“The incessant equilibration of constant divergences is accomplished so much more quickly, 1) the more mobile the capital, i.e., the more easily it can be shifted from one sphere and from one place to another; 2) the more quickly labour-power can be transferred from one sphere to another and from one production locality to another.” -Vol. 3, p. 143

Just as the rate of exploitation is equalized by the perfect mobility of labor (see Result 2 in this post), the rate of profit is equalized by perfect capital mobility.

In the model of perfect competition, mainstream microeconomics also maintains an assumption of perfect capital mobility, which is thought to drive the rate of profit down to zero for all industries. Marxian economics thus agrees with the premise of capital mobility, but disagrees with the conclusion that profit will be driven down to zero, for if it were then in Morishima’s words “capitalists would not be interested in their enterprises.” In Marx’s words from Volume I:

“Use-values must therefore never be looked upon as the real aim of the capitalist; neither must the profit on any single transaction. The restless never-ending process of profit-making alone is what he aims at. This boundless greed after riches, this passionate chase after exchange-value, is common to the capitalist and the miser; but while the miser is merely a capitalist gone mad, the capitalist is a rational miser. The never-ending augmentation of exchange-value, which the miser strives after, by seeking to save his money from circulation, is attained by the more acute capitalist, by constantly throwing it afresh into circulation.” -Vol. 1, p. 107

---

“The directing motive, the end and aim of capitalist production, is to extract the greatest possible amount of surplus-value, and consequently to exploit labour-power to the greatest possible extent.” - Vol. 1, p. 231

With such a strong profit motive it certainly stands to reason that capitalists would exit fields of industry in which positive rates of profit were not attainable.

In any event, replacing $\pi_i$ with the common rate of profit $\pi$ in equations (4c) and (5c) yields what we refer to as the “price-determining equations”, i.e., the equations satisfied by the equilibrium prices $p_i$ ($i=1,\ldots,m$) and rate of profit $\pi$:

\[
p_i = (1 + \pi)(p_i \text{col}_i A_i + w l_i) \quad \text{for } i=1,\ldots,n \quad (6a) \\
p_i = (1 + \pi)(p_i \text{col}_{i-n} A_{II} + w l_i) \quad \text{for } i=n+1,\ldots,m \quad (6b)
\]

There is a statement in Volume 3 that bears a strong resemblance to (6a) and (6b):

“The formula that the price of production of a commodity = k + p, i.e., equals its cost-price plus profit, is now more precisely defined with $p = kp'$ ($p'$ being the general rate of profit). Hence the price of production = $k + kp'$.” Vol. 3, p. 125

If we equate the price of production with the market price of the good ($p_i$) and the cost-price $k$ to the cost of the labor and capital inputs ($p_i \text{col}_i A_i + w l_i$) and the general rate of profit $p'$ with $\pi$, then (6a) and (6b) are an exact match with the equation [price of production = $(1+ p')k$]. This suggests the approach we have taken thus far to lead up to (6a) and (6b), including defining profits as in (2a) and (2b), the rate of profit as profits over production costs, and assuming a uniform rate of profit $\pi$, is consistent with Marx’s own conception of these topics.

We now proceed to show that under the subsistence-wage assumption, the price-determining equations (6a) and (6b) can be written in the vector form $p = (1 + \pi)pM$, where $p$ is the vector of prices of all goods and $M$ is the input power matrix. Stacking the equations (6a) for capital goods side-by-side into a (1 x $n$) vector equation, we get:

\[
[p_1 \quad p_2 \quad \cdots \quad p_n] = [(1 + \pi)p_1 \text{col}_1 A_1 \quad (1 + \pi)p_1 \text{col}_2 A_1 \quad \cdots \quad (1 + \pi)p_1 \text{col}_n A_1] \\
\quad + [(1 + \pi)wl_1 \quad (1 + \pi)wl_2 \quad \cdots \quad (1 + \pi)wl_n] \quad (7a)
\]

\[
\Rightarrow p_i = (1 + \pi)p_1 [\text{col}_1 A_1 \quad \text{col}_2 A_1 \quad \cdots \quad \text{col}_n A_1] + (1 + \pi)w[l_1 \quad l_2 \quad \cdots \quad l_n] \quad (7b)
\]

\[
\Rightarrow p_i = (1 + \pi)p_1 A_i + (1 + \pi)wl_i \quad (7c)
\]

\[
\Rightarrow p_i = (1 + \pi)[p_i A_i + wL_i], \quad (7d)
\]

where $L_i$ is the labor input vector for capital goods. Doing the same with equations (6b) for wage and luxury goods, we get a (1 x ($m-n$)) vector equation:

\[
[p_{n+1} \quad p_{n+2} \quad \cdots \quad p_m] = [(1 + \pi)p_1 \text{col}_1 A_{II} \quad (1 + \pi)p_1 \text{col}_2 A_{II} \quad \cdots \quad (1 + \pi)p_1 \text{col}_{m-n} A_{II}] \\
\quad + [(1 + \pi)wl_{n+1} \quad (1 + \pi)wl_{n+2} \quad \cdots \quad (1 + \pi)wl_m] \quad (8a)
\]

\[
\Rightarrow p_{II} = (1 + \pi)p_1 [\text{col}_1 A_{II} \quad \text{col}_2 A_{II} \quad \cdots \quad \text{col}_{m-n} A_{II}] + (1 + \pi)w[l_{n+1} \quad l_{n+2} \quad \cdots \quad l_m] \quad (8b)
\]
\[ p_I = (1 + \pi)[p_I A_{II} + p_{II} \omega B L_{II}] \]
\[ p_{II} = (1 + \pi)[p_I A_{II} + wL_{II}], \]
\[ \Rightarrow p_{II} = (1 + \pi)[p_I A_{II} + wL_{II}], \]
\[ \Rightarrow p_{II} = (1 + \pi)[p_I A_{II} + p_{II} \omega B L_{II}], \]
\[ \Rightarrow p = (1 + \pi)pM, \]

where \( L_{II} \) is the labor input vector for wage and luxury goods. Assuming a subsistence wage, i.e., that the wage is equal to the cost of the hourly subsistence bundle of goods, we can replace \( w \) with \( p_{II} \omega B \) in (7d) and (8d) to obtain:

\[ p_I = (1 + \pi)[p_I A_I + p_{II} \omega B L_I] \]
\[ p_{II} = (1 + \pi)[p_I A_{II} + p_{II} \omega B L_{II}] \]

Now stacking (9a) and (9b) side-by-side into one big (1 x m) vector equation, we obtain:

\[ \begin{bmatrix} p_I \\ p_{II} \end{bmatrix} = (1 + \pi) \begin{bmatrix} p_I A_I + p_{II} \omega B L_I \\ p_I A_{II} + p_{II} \omega B L_{II} \end{bmatrix} \]
\[ \Rightarrow \begin{bmatrix} p_I \\ p_{II} \end{bmatrix} = (1 + \pi) \begin{bmatrix} p_I A_I + p_{II} \omega B L_I \\ p_I A_{II} + p_{II} \omega B L_{II} \end{bmatrix} \]
\[ \Rightarrow \begin{bmatrix} p_I \\ p_{II} \end{bmatrix} = (1 + \pi) \begin{bmatrix} A_I & A_{II} \\ \omega B L_I & \omega B L_{II} \end{bmatrix} \]
\[ \Rightarrow p = (1 + \pi)pM, \]

where \( p \) is the vector of prices for all goods and \( M = \begin{bmatrix} A_I & A_{II} \\ \omega B L_I & \omega B L_{II} \end{bmatrix} \) is the input power matrix.

This establishes that the price determining equations (6a) and (6b) are equivalent to the vector equation \( p = (1 + \pi)pM \). Thus, if desired, we can refer to the latter as the price-determining equation as well. Note also that, for the capitalist system to be viable, the common rate of profit would have to be positive, so \( \pi > 0 \) would have to hold in (10d).

Note that the price-determining equation can alternatively be written as \( pM = \mu p \), where \( \mu = (1 + \pi)^{-1} \). This means that pre-multiplying \( M \) by \( p \) gives us a vector that is a constant multiple of \( p \), where that multiple is \( \mu \). The value \( \mu \) is called an “eigenvalue” of \( M \) and the vector \( p \) is an “eigenvector” (in this case, it is a “left eigenvector” because \( M \) is multiplied by \( p \) on the left) associated with the eigenvalue \( \mu \). We find eigenvalues of an \((n \times n)\) real matrix \( A \) by solving the equation \(|A - \mu I| = 0\) where \( I \) is the \((n \times n)\) identity matrix; that is, by solving for the values of \( \mu \) that make the determinant of \( A - \mu I \) zero. In general, this will be an \( n^{th} \) degree polynomial with real number coefficients. By the Fundamental Theorem of Algebra, such an equation will have exactly \( n \) complex roots if we count multiplicities (which means, if \(-1+2i\) is a root of the equation 3 times, we count each of these appearances as distinct). This means that an \((n \times n)\) square matrix \( A \) will have (up to multiplicities) \( n \) complex eigenvalues. Some of these eigenvalues may be real and some may be truly complex. Having obtained an eigenvalue \( \mu \) for a matrix \( A \), we find a (left) eigenvector associated with \( \mu \) by solving the equation \( xA = \mu x \). Eigenvectors for a given eigenvalue are never unique, for if \( xA = \mu x \) then for any number \( \alpha \) we also have \( \alpha xA = \mu \alpha x \), so that if \( x \) is an eigenvector for \( \mu \), then \( \alpha x \) is as well. When solving for eigenvectors we could pick

---

4 Note that, if it were not possible to identify a non-negative rate of profit with a subsistence wage, it would certainly not be possible to do so with a higher wage. Thus the subsistence wage assumption gives us the best possible chance to establish the existence of a non-negative rate of profit.

5 A complex number is a number that can be written in the form \( z = a + bi \), where \( a \) and \( b \) are real numbers and the “imaginary number” \( i \) is defined as \( i = \sqrt{-1} \). Thus any real number \( r \) is also a complex number because we can write it in the form \( r = r + 0i \). The complex number \( z = a + bi \) can be identified with the point in the two-dimensional plane having coordinates \((a, b)\). In a similar fashion the real numbers can be identified with the x-axis of the plane.
out a unique one by imposing some kind of condition on it; a common one is that its length (the square root of the sum of squares of its entries) be equal to 1. If we could somehow know that the matrix $M$ had a real eigenvalue $\mu$, then the value $(1/\mu) - 1$ would be a candidate for the rate of profit, and any associated left eigenvector $p$ would be a candidate for the equilibrium vector of prices. Of course, for the rate of profit to be non-negative, $\mu$ would need to be no larger than 1, and for the prices of all goods to be strictly positive, the eigenvector would need to be strictly positive. Even better would be if we could somehow pick out a unique eigenvalue that’s less than or equal to 1; its reciprocal minus 1 could serve as the rate of profit; the price vector can be obtained by imposing a condition to pick out a particular strictly positive eigenvector (e.g., we could designate one of the goods in the economy as numeraire and arbitrarily set its price equal to 1; then the eigenvector with 1 in this particular entry would be the equilibrium price vector). We now show that, under Assumption Blocks 1, 4, and 5, it is possible to identify a particular eigenvalue less than or equal to 1 that has a strictly positive associated eigenvector. Note that under Assumption Blocks 1 and 4, $M$ is a non-negative matrix. Under Assumption Block 5, it is indecomposable (or “irreducible” in the language of linear algebra). Since it is also square (i.e., $m \times m$), the Perron–Frobenius Theorem⁶ implies that $M$ has a positive real eigenvalue $r$ such that: (i) any other eigenvalue of $M$ has complex modulus⁷ less than or equal to $r$, (ii) the left and right eigenspaces associated with $r$ are one-dimensional (i.e., any two eigenvectors associated with $r$ must be constant multiples of each other), and (iii) there is a strictly positive right eigenvector $v$ and a strictly positive left eigenvector $w$ associated with $r$.

The “spectral radius” of $M$ is the maximum complex modulus among all the eigenvalues of $M$. By above implication (i) of the Perron–Frobenius Theorem, the spectral radius of $M$ is $r$. Under Assumption Block 5, we have $r \leq 1$. Let $\pi = (1/r) - 1$, which is $\geq 0$. Note that $\pi$ is a well-defined value because $r$ is a uniquely-defined number associated with $M$. Under the above implication (iii), there exists a strictly positive vector $w$ such that $wM = rw$. Let $w_{n+1}$ denote the $(n+1)\text{st}$ entry of $w$. Since $w_{n+1} > 0$, we can let $p = w_{n+1}^{-1}w$. Note that $p$ is a left eigenvector for $M$ because it is a constant multiple of $w$. Also, $p$ is strictly positive and its $(n+1)\text{st}$ entry is 1. In fact, $p$ is the unique left eigenvector of $M$ whose $(n+1)\text{st}$ entry is 1. To see this, suppose that $x$ is any left eigenvector of $M$ whose $(n+1)\text{st}$ entry is 1. Then since the left eigenspace associated with $r$ is one-dimensional by the above implication (ii), we have $x = \alpha p$ for some number $\alpha$, so that in particular $x_{n+1} = \alpha p_{n+1}$ where $x_{n+1}$ and $p_{n+1}$ are the $(n+1)\text{st}$ entries of the vectors $x$ and $p$, respectively. But since $x_{n+1} = 1$ and $p_{n+1} = 1$, this becomes $1 = \alpha$, so that $x = p$. Hence any left eigenvector with 1 as its $(n+1)\text{st}$ entry is in fact $p$. This means $p$ is unique. Essentially we have chosen good $(n+1)$, the first wage/luxury good, to be the numeraire and chosen as the price vector the left eigenvector of $r$ whose entries are consistent with this choice of numeraire. The fact that $p$ is a left eigenvector for $M$ means that $pM = rp$, which implies $p = r^{-1}pM$ or $p = (1+\pi)pM$ by definition of $\pi$. Thus we have found a unique number $\pi \geq 0$ and a unique vector

---


⁷ The “complex modulus” or “absolute value” of the complex number $z=a+bi$ is defined as $|z|=\sqrt{a^2+b^2}$. This is the distance in the two-dimensional plane between the point $(a,b)$, which we identify with $z$, and the origin $(0,0)$. Thus the modulus is the distance between $z$ and the origin. In the case of a real number, the complex modulus is just the usual definition of the absolute value. It follows that the Perron-Frobenius Theorem says that for a non-negative irreducible square matrix, there exists an eigenvalue of maximal absolute value, and this eigenvalue is real.
\( p > 0 \) such that the price-determining equations are satisfied, which means a unique capitalist equilibrium exists.

---

**DISCUSSION**

Since we have chosen the rate of profit to be the reciprocal of the maximal eigenvalue (minus 1), we have in essence selected the smallest possible rate of profit that is consistent with both non-negative profits and the price-determining equations. This can be thought of as a kind of free-entry assumption for this Marxian model. The free mobility of capital (an idea which Marx clearly believed in as described above) drives the rate of profit down as low as it can possibly be while still sustaining the capitalist mode of production. However, unlike the general equilibrium models with perfect competition that are pervasive in modern microeconomics, we do not insist that this minimal rate of profit is zero. In general it will be positive, consistent with Marx’s view that capitalists would not continue in their businesses if they could not earn positive profits. The rate of profit will only be zero in the case that the spectral radius of \( M \) is exactly 1. If the spectral radius of \( M \) turns out to exceed 1, then the rate of profit selected according to the above process is negative. We interpret this to mean that the input power matrix is unable to sustain a capitalist equilibrium.